Abductive Question-Answer System (AQAS) for Classical Propositional Logic

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Reasoning 2022/2023

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Outline

Abductive reasoning

The surprising fact, C, is observed.
But if A were true, C would be a matter of course.

Hence, there is reason to suspect that A is true. $\therefore$

The street is wet.
If it rained then the streets would be wet.

It rained. $\therefore$

But maybe the snow melted...

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2. H — an abductive hypothesis;
   ϕ can be computed/derived from Γ' which is equal to Γ augmented with H i.e., $\Gamma \cup \{H\} \models \phi$.

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   Hintikka sets and dual Hintikka sets/...

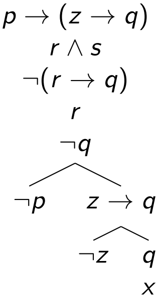
Analytic Tableaux

α	α_1	α_2	β	β_1	β_2
$A \wedge B$	A	B	$\neg(A \wedge B)$	$\neg A$	$\neg B$
$\neg(A \vee B)$	$\neg A$	$\neg B$	$A \vee B$	A	B
$\neg(A \to B)$	A	$\neg B$	$A \to B$	$\neg A$	B

$$\begin{array}{c} \alpha \\ | \\ \alpha_1 \\ \alpha_2 \end{array} \qquad \begin{array}{c} \beta \\ \diagup \quad \diagdown \\ \beta_1 \quad \beta_2 \end{array} \qquad \begin{array}{c} \neg\neg A \\ | \\ A \end{array}$$

Example

Let $\Gamma = \{p \to (z \to q), r \wedge s\}$ and $A = r \to q$.



Hypotheses: $p \wedge z$, $p \wedge q$, q, $\neg r$, ...

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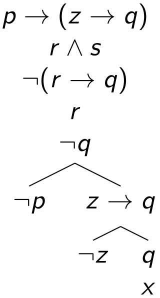

Hypotheses: $p \wedge z$, $p \wedge q$, q, $\neg r$, $\ldots$
Which of them are good?

Properties of abductive hypotheses

Properties of abductive hypotheses

Γ = knowledge base, A = abductive goal, H = hypothesis

- consistency: $\Gamma \cup \{H\}$ is consistent;
- significance: $H \nvdash A$;
- complexity: *simpler* hypotheses are better;
- minimality: weaker hypotheses vs stronger ones — if p is good abductive hypothesis, then $p \wedge q$ seems to strong.

Find abductive hypotheses for the following problems. What properties do they have?

1. $\Gamma = \{q \vee r, \neg q\}$, $A = s$
2. $\Gamma = \{p \rightarrow q, q \rightarrow r, r \rightarrow s\}$, $A = s$
3. $\Gamma = \{p \rightarrow q, r \vee s\}$, $A = p \rightarrow s$
4. $\Gamma = \{(p \vee q) \rightarrow r, s \rightarrow p, t \rightarrow q\}$, $A = r$

Abductive Question-Answer System

Abductive Question-Answer System

Rules for
questions procesing

$$\frac{Question}{Question}$$

Rules for
answering questions

$$\frac{Question}{Answer}$$

Questions

Questions of $\mathcal{L}^{?}_{CPL}$

An atomic declarative formula (sequent) of $\mathcal{L}^{?}_{\mathsf{CPL}}$

$$\Gamma \vdash \Delta$$

where Γ and Δ are finite, non-empty, sequences of formulas of $\mathcal{L}_{\mathsf{CPL}}$.

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Questions of $\mathcal{L}^?_{\mathsf{CPL}}$

$$?(\Phi)$$

where Φ is a finite, non-empty sequence of sequents of $\mathcal{L}^?_{\mathsf{CPL}}$.

Rules for processing questions of $\mathbb{E}^{\mathsf{CPL}}$

$$\frac{?(\Phi; \Gamma, \alpha, \Gamma' \vdash \Delta; \Psi)}{?(\Phi; \Gamma, \alpha_1, \alpha_2, \Gamma' \vdash \Delta; \Psi)}\ \mathbf{L}_\alpha \qquad \frac{?(\Phi; \Gamma \vdash \Delta, \alpha, \Delta'; \Psi)}{?(\Phi; \Gamma \vdash \Delta, \alpha_1, \Delta'; \Gamma \vdash \Delta, \alpha_2, \Delta'; \Psi)}\ \mathbf{R}_\alpha$$

$$\frac{?(\Phi; \Gamma, \beta, \Gamma' \vdash \Delta; \Psi)}{?(\Phi; \Gamma, \beta_1, \Gamma' \vdash \Delta; \Gamma, \beta_2, \Gamma' \vdash \Delta; \Psi)}\ \mathbf{L}_\beta \qquad \frac{?(\Phi; \Gamma \vdash \Delta, \beta, \Delta'; \Psi)}{?(\Phi; \Gamma \vdash \Delta, \beta_1, \beta_2, \Delta'; \Psi)}\ \mathbf{R}_\beta$$

$$\frac{?(\Phi; \Gamma, \neg\neg A, \Gamma' \vdash \Delta; \Psi)}{?(\Phi; \Gamma, A, \Gamma' \vdash \Delta; \Psi)}\ \mathbf{L}_{\neg\neg} \qquad \frac{?(\Phi; \Gamma \vdash \Delta, \neg\neg A, \Delta'; \Psi)}{?(\Phi; \Gamma \vdash \Delta, A, \Delta'; \Psi)}\ \mathbf{R}_{\neg\neg}$$

$$\frac{\alpha_1, \alpha_2, \Gamma \vdash \Delta}{\alpha, \Gamma \vdash \Delta}\ \mathbf{L}_\alpha \qquad \frac{\Gamma \vdash \Delta, \alpha_1 \quad \Gamma \vdash \Delta, \alpha_2}{\Gamma \vdash \Delta, \alpha}\ \mathbf{R}_\alpha$$

$$\frac{\beta_1, \Gamma \vdash \Delta \quad \beta_2, \Gamma \vdash \Delta}{\beta, \Gamma \vdash \Delta}\ \mathbf{L}_\beta \qquad \frac{\Gamma \vdash \Delta, \beta_1, \beta_2}{\Gamma \vdash \Delta, \beta}\ \mathbf{R}_\beta$$

$$\frac{A, \Gamma \vdash \Delta}{\neg\neg A, \Gamma \vdash \Delta}\ \mathbf{L}_{\neg\neg} \qquad \frac{\Gamma \vdash \Delta, A}{\Gamma \vdash \Delta, \neg\neg A}\ \mathbf{R}_{\neg\neg}$$

Rules for processing questions of $\mathbb{E}^{\mathsf{CPL}}$

Socratic transformation

A finite sequence of questions $\mathbf{s} = \langle s_1, \ldots, s_n \rangle$ is a *Socratic transformation* (s-transformation) of the question $?(\Phi)$ by means of $\mathbb{E}^{\mathsf{CPL}}$ iff the following conditions hold:

- $s_1 = ?(\Phi)$,
- s_i results from s_{i-1} (where $i > 1$) by an application of a rule of $\mathbb{E}^{\mathsf{CPL}}$.

Example

Knowledge base:

$$\Gamma = \langle p \rightarrow (q \rightarrow r), \neg(q \rightarrow s) \rangle$$

Example

Knowledge base:

$$\Gamma = \langle p \to (q \to r), \neg(q \to s)\rangle$$

What we want to derive:

$$\Delta = \langle z \rangle$$

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What we want to derive:

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The question arises:

$$?(\Gamma \vdash \Delta)$$

$$?(p \rightarrow (q \rightarrow r), \neg(q \rightarrow s) \vdash z)$$

Example

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

Example

$$\cfrac{\cfrac{\cfrac{?(p \to (q \to r), \neg(q \to s) \vdash z)}{?(p \to (q \to r), q, \neg s \vdash z)}\ \mathbf{L}_\alpha}{?(\neg p, q, \neg s \vdash z\ ;\ q \to r, q, \neg s \vdash z)}\ \mathbf{L}_\beta}{?(\neg p, q, \neg s \vdash z\ ;\ \neg q, q, \neg s \vdash z\ ;\ r, q, \neg s \vdash z)}\ \mathbf{L}_\beta$$

Produce Socratic transformations of the following questions:

1. $?(\neg(\neg p \wedge q) \vdash p \vee \neg\neg\neg q)$
2. $?((p \to q) \to p \vdash p)$
3. $?(p \vee q \vdash (p \to r) \to ((q \to r) \to r))$
4. $?(p \to q, q \to p, \neg(p \wedge r) \vdash \neg q \vee \neg r)$

Answers

How to answer an abductive question?

To answer an abductive question $?(\Gamma \vdash \Delta)$ we employ the following procedure:

Step 1 Create a complete s-transformation of the question $?(\Gamma \vdash \Delta)$; the last question of this s-transformation is based on a sequence of sequents each of which consists of literals only.

Step 2 Apply some abductive rules (to be introduced later on) to this last question; each rule is *local* in the sense that only one sequent at a time is active in such a rule.

Step 3 Combine the results of the applications of rules using a conjunction; the resulting hypothesis has the form $H = A_1 \wedge \ldots \wedge A_n$, where each A_i $(1 \leq i \leq n)$ is the conclusion of an abductive rule.

By a *literal* we will mean a propositional variable or negation of a propositional variable.
Moreover, if $l = p$ then $\bar{l} = \neg p$ and if $l = \neg p$ then $\bar{l} = p$.

$$\frac{?(\Phi\ ;\ \Theta, l, \Theta' \vdash \Theta''\ ;\ \Psi)}{\bar{l}}\ \mathbf{R}^1_{abd}$$

$$\frac{?(\Phi\ ;\ \Theta, l, \Theta' \vdash \Theta'', k, \Theta'''\ ;\ \Psi)}{l \to k}\ \mathbf{R}^2_{abd}$$

Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

$$\vdots$$

$$?(\ \neg p, q, \neg s \vdash z\ ;\ \neg q, q, \neg s \vdash z\ ;\ r, q, \neg s \vdash z\ )$$

For $\neg p, q, \neg s \vdash z$:

$\mathbf{R}^1_{abd}$: p, $\neg q$, s

$\mathbf{R}^2_{abd}$: $\neg p \to z$, $q \to z$, $\neg s \to z$

Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

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For $\neg p, q, \neg s \vdash z$:

$\mathbf{R}^1_{abd}$: $p,\ \neg q,\ s$

$\mathbf{R}^2_{abd}$: $\neg p \to z,\ q \to z,\ \neg s \to z$

For $r, q, \neg s \vdash z$:

$\mathbf{R}^1_{abd}$: $\neg r,\ \neg q,\ s$

$\mathbf{R}^2_{abd}$: $r \to z,\ q \to z,\ \neg s \to z$

Properties of abductive hypotheses

Properties of abductive hypotheses

1. Consistency: $\Gamma \cup \{H\}$ is consistent.
2. Significance: $H \nvdash_{\mathsf{CPL}} A$.

Downward saturated set

Let Γ be a sequence of formulas of $\mathcal{L}_{\mathsf{CPL}}$. By a *downward saturated set* (or *Hintikka set*) corresponding to a sequence Γ we mean a set $\mathfrak{U}_\Gamma$, which fulfils the following conditions:

1. if A is a term of Γ, then $A \in \mathfrak{U}_\Gamma$,
2. if $\alpha \in \mathfrak{U}_\Gamma$, then $\alpha_1 \in \mathfrak{U}_\Gamma$ and $\alpha_2 \in \mathfrak{U}_\Gamma$,
3. if $\beta \in \mathfrak{U}_\Gamma$, then $\beta_1 \in \mathfrak{U}_\Gamma$ or $\beta_2 \in \mathfrak{U}_\Gamma$,
4. if $\neg\neg A \in \mathfrak{U}_\Gamma$, then $A \in \mathfrak{U}_\Gamma$.
5. nothing more belongs to $\mathfrak{U}_\Gamma$ except those formulas which enter $\mathfrak{U}_\Gamma$ on the grounds of conditions 1–4.

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$$\mathfrak{U}_\Gamma^c = \{\mathfrak{U}_\Gamma^1, \ldots, \mathfrak{U}_\Gamma^n\}$$

Propertiec of abductive hypotheses

$$\frac{?(\Phi \ ; \ \Theta, l, \Theta' \vdash \Theta'' \ ; \ \Psi)}{\bar{l}} \ \mathbf{R}^1_{abd}$$

There exists a set $\mathfrak{U}_\Gamma \in \mathfrak{U}^c_\Gamma$ such that

$$l \notin \mathfrak{U}_\Gamma$$

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$$\frac{?(\Phi\ ;\ \Theta, l, \Theta' \vdash \Theta'', k, \Theta'''\ ;\ \Psi)}{l \rightarrow k}\ \mathbf{R}^2_{abd}$$

There exists a set $\mathfrak{U}_\Gamma \in \mathfrak{U}^c_\Gamma$ such that

$$l \notin \mathfrak{U}_\Gamma \quad \text{or} \quad \overline{k} \notin \mathfrak{U}_\Gamma$$

Dual downward saturated set

Let Δ be a sequence of formulas of $\mathcal{L}_{\mathsf{CPL}}$. By a *dual downward saturated set* (or *dual Hintikka set*) corresponding to a sequence Δ we mean a set $\mathfrak{W}_\Delta$, which fulfils the following conditions:

1. if A is a term of Δ, then $A \in \mathfrak{W}_\Delta$,
2. if $\alpha \in \mathfrak{W}_\Delta$, then $\alpha_1 \in \mathfrak{W}_\Delta$ or $\alpha_2 \in \mathfrak{W}_\Delta$,
3. if $\beta \in \mathfrak{W}_\Delta$, then $\beta_1 \in \mathfrak{W}_\Delta$ and $\beta_2 \in \mathfrak{W}_\Delta$,
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5. nothing more belongs to $\mathfrak{W}_\Delta$ except those formulas which enter $\mathfrak{W}_\Delta$ on the grounds of conditions 1–4.

$$\mathfrak{W}^{nv}_\Delta = \{\mathfrak{W}^1_\Delta, \ldots, \mathfrak{W}^n_\Delta\}$$

$$\frac{?(\Phi\ ;\ \Theta, l, \Theta' \vdash \Theta''\ ;\ \Psi)}{\bar{l}}\ \mathbf{R}^1_{abd}$$

There exists a set $\mathfrak{W}_\Delta \in \mathfrak{W}^{nv}_\Delta$ such that

$$\bar{l} \notin \mathfrak{W}_\Delta$$

Properties of abductive hypotheses

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$$\frac{?(\Phi\ ;\ \Theta, l, \Theta' \vdash \Theta'', k, \Theta'''\ ;\ \Psi)}{l \to k}\ \mathbf{R}^2_{abd}$$

There exists a set $\mathfrak{W}_\Delta \in \mathfrak{W}^{nv}_\Delta$ such that

$$\bar{l} \notin \mathfrak{W}_\Delta \quad \text{or} \quad k \notin \mathfrak{W}_\Delta$$

Consistent abductive hypotheses

1. Choose A_i.
2. Leave in $\mathfrak{U}_\Gamma^c$ only those $\mathfrak{U}_\Gamma$ that are consistent with A_i.
3. If there are still open sequents, then choose A_j...

$\Gamma = \langle p \to (q \to r), \neg(q \to s)\rangle$

$\Delta = \langle z \rangle$

$$\dfrac{\dfrac{\dfrac{?(p \to (q \to r), \neg(q \to s) \vdash z)}{?(p \to (q \to r), q, \neg s \vdash z)}\ \mathbf{L}_\alpha}{?(\neg p, q, \neg s \vdash z\ ;\ \ q \to r, q, \neg s \vdash z)}\ \mathbf{L}_\beta}{?(\neg p, q, \neg s \vdash z\ ;\ \ \neg q, q, \neg s \vdash z\ ;\ \ r, q, \neg s \vdash z)}\ \mathbf{L}_\beta$$

$$\Gamma = \langle p \to (q \to r), \neg(q \to s) \rangle$$

$$\mathfrak{U}^1_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, \neg p\}$$

$$\mathfrak{U}^2_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, q \to r, \neg q\}$$

$$\mathfrak{U}^3_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, q \to r, r\}$$

$$\mathfrak{U}^4_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, q \to r, r, \neg q\}$$

$$\mathfrak{U}^5_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, q \to r, r, \neg q, \neg p\}$$

$$\mathfrak{U}^6_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, q \to r, r, \neg p\}$$

$$\mathfrak{U}^7_\Gamma = \{p \to (q \to r), \neg(q \to s), q, \neg s, q \to r, \neg q, \neg p\}$$

$$\mathfrak{U}^c_\Gamma = \{\mathfrak{U}^1_\Gamma, \mathfrak{U}^3_\Gamma, \mathfrak{U}^6_\Gamma\}$$

Example — continuation

$$\Delta = \langle z \rangle$$

$$\mathfrak{W}^1_\Delta = \{z\}$$

$$\mathfrak{W}^{nv}_\Delta = \{\mathfrak{W}^1_\Delta\}$$

Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

$$\vdots$$

$$?(\ \neg p, q, \neg s \vdash z\ ;\ \neg q, q, \neg s \vdash z\ ;\ r, q, \neg s \vdash z\ )$$

$\mathfrak{U}^c_\Gamma$:

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$A_1 = p$ by means of $\mathbf{R}^1_{abd}$

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$A_1 = p$ by means of $\mathbf{R}^1_{abd}$

$A_2 = r \to z$ by means of $\mathbf{R}^2_{abd}$

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Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

$$\vdots$$

$$?(\neg p, q, \neg s \vdash z \; ; \; \neg q, q, \neg s \vdash z \; ; \; r, q, \neg s \vdash z)$$

$A_1 = p$ by means of $\mathbf{R}^1_{abd}$

$A_2 = r \to z$ by means of $\mathbf{R}^2_{abd}$

$$H = p \wedge (r \to z)$$

$\mathfrak{U}^c_\Gamma$:

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Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

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Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

$$\vdots$$

$$?(\boxed{\neg p, q, \neg s \vdash z}\ ;\ \neg q, q, \neg s \vdash z\ ;\ \boxed{r, q, \neg s \vdash z})$$

$H = p \wedge (r \to z)$

Example — continuation

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$H = p \wedge (r \to z)$

$p \wedge (r \to z), p \to (q \to r), \neg(q \to s) \vdash_{\mathrm{CPL}} z$

Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

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$p \wedge (r \to z), p \to (q \to r), \neg(q \to s) \vdash_{\mathrm{CPL}} z$

$p \wedge (r \to z), p \to (q \to r), \neg(q \to s) \nvdash_{\mathrm{CPL}} \bot$ consistency

Example — continuation

$$?(p \to (q \to r), \neg(q \to s) \vdash z)$$

$$\vdots$$

$$?(\neg p, q, \neg s \vdash z \; ; \; \neg q, q, \neg s \vdash z \; ; \; r, q, \neg s \vdash z)$$

$H = p \wedge (r \to z)$

$p \wedge (r \to z), p \to (q \to r), \neg(q \to s) \vdash_{\mathsf{CPL}} z$

$p \wedge (r \to z), p \to (q \to r), \neg(q \to s) \nvdash_{\mathsf{CPL}} \bot$ consistency

$p \wedge (r \to z) \nvdash_{\mathsf{CPL}} z$ significance

Using s-transformations, Hintikka and dual Hintikka sets find consistent and significant abductive hypotheses for the following problems:

1. $?(\neg(\neg p \wedge q) \vdash \neg\neg\neg q)$
2. $?(p \vee q \vdash (q \rightarrow r) \rightarrow r)$
3. $?(p \rightarrow q, q \rightarrow p, \neg(p \wedge r) \vdash \neg q)$
4. $?(p \vee q, q \rightarrow r \vdash r \vee \neg q)$
5. $?(p \rightarrow q, q \rightarrow r, r \rightarrow s \vdash s)$
6. $?(q \vee r, \neg q \vdash s)$
7. $?(p \rightarrow q, r \vee s \vdash p \rightarrow s)$
8. $?((p \vee q) \rightarrow r, s \rightarrow p, t \rightarrow q \vdash r)$

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$$\frac{?(\Phi \ ; \ \Theta, l, \Theta' \vdash \Theta'' \ ; \ \Psi)}{\bar{l}} \ \mathbf{R}^1_{abd}$$

$$\frac{?(\Phi \ ; \ \Theta, l, \Theta' \vdash \Theta'', k, \Theta''' \ ; \ \Psi)}{l \rightarrow k} \ \mathbf{R}^2_{abd}$$

$$\frac{?(\Phi\ ;\ \Theta, \chi \circ p, \Theta', \chi \sim p, \Theta'' \vdash \Theta'''\ ;\ \Psi)}{p}\ \mathbf{R}^3_{abd}$$

$$\frac{?(\Phi\ ;\ \Theta, \chi \circ p, \Theta', p, \Theta'' \vdash \Theta'''\ ;\ \Psi)}{\sim p}\ \mathbf{R}^4_{abd}$$

$$\frac{?(\Phi\ ;\ \Theta, p, \Theta', \chi \sim p, \Theta'' \vdash \Theta'''\ ;\ \Psi)}{\circ p}\ \mathbf{R}^5_{abd}$$

$$\frac{?(\Phi \;;\; \Theta, \chi \sim p, \Theta', \chi \circ p, \Theta'' \vdash \Theta''' \;;\; \Psi)}{p} \;\; \mathbf{R}^{3^*}_{abd}$$

$$\frac{?(\Phi \;;\; \Theta, p, \Theta', \chi \circ p, \Theta'' \vdash \Theta''' \;;\; \Psi)}{\sim p} \;\; \mathbf{R}^{4^*}_{abd}$$

$$\frac{?(\Phi \;;\; \Theta, \chi \sim p, \Theta', p, \Theta'' \vdash \Theta''' \;;\; \Psi)}{\circ p} \;\; \mathbf{R}^{5^*}_{abd}$$

$$\frac{?(\Phi \ ; \ \Theta, l, \Theta' \vdash \Theta'' \ ; \ \Psi)}{\bar{l}} \ \mathbf{R}^1_{abd}$$

$$\frac{?(\Phi \ ; \ \Theta, l, \Theta' \vdash \Theta'', k, \Theta''' \ ; \ \Psi)}{l \rightarrow k} \ \mathbf{R}^2_{abd}$$

$$\frac{?(\Phi\ ;\ x_1 R \cdots R x_n, \Theta, x_n : l, \Theta' \vdash \Theta''\ ;\ \Psi)}{\Box_{n-1}\bar{l}}\ \mathbf{R}^{1m}_{abd}$$

$$\frac{?(\Phi\ ;\ x_1 R \cdots R x_n, \Theta, x_i : l, \Theta' \vdash \Theta'', x_n : k, \Theta'''\ ;\ \Psi)}{\Box_{i-1}(l \to \Box_{n-i} k)}\ \mathbf{R}^{2m}_{abd}$$